Demographic Methodology

Updated May 24, 2019

Contents

Methods for creating the life table

Data are imported to PMx from multiple sources, the two most common being the PopLink csv file, SPARKS 1.6 Exchange.csv file. In both these exports, the data come in a format basically providing the following demographic data for each individual that meets the export filter criteria:

Demographic Individual Information

- ID
- Sex
- Date of Birth
- Date of Birth Estimate
- Dam(s)
- Sire(s)
- Date of Death
- Date of Death Estimate

Individual's events relative to the risk window:

- ID
- DateIn Risk Window
- DateIn Estimate
- Intype (Alive, Birth, Imported)
- DateOut of Risk Window
- DateOut Estimate
- OutType (Alive, Death, LTF)

The Intype and Outtype codes differ between PopLink and SPARKS.
The following conditions exclude any individual from the analysis:

- unknown date for any of the date events
- dates out of order (e.g., birth date after Date in; Death date before DateIn)
- those that are "MateOf": PMx-created individuals that are undefined individuals but parents of individuals that are in the dataset;
- InType or OutType unknown.

The age in days of the individual at DateIn and DateOut is calculated using the dates of Birth, DateOut and DateIn ignoring the date estimated associated with those dates. The events in a Frequency Data Table (Table X) are then incremented according to the age in days of the event and InType and OutType. A frequency data table is derived for both females and males. Individuals of sex other than male or female are default treated as .5 male and .5 female. This 50/50 assignment can be changed in Demographic Settings Tab:

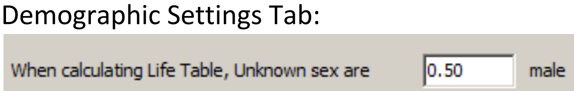


Table X.

i	Age(Days)	AliveIn	AliveOut	Born	Arrived	Died	Left	LTF	Reproduced	Risk
0	0	0	0	192	15.5	6	4.5	0	0	192
1	1	0	0	0	1	4	0	0	0	197

2	2	0	0	0	0	5.5	0	0	0	194
3	3	0	0	0	0	1.5	0	0	0	188.5
4	6	0	0	0	0	3	1	0	0	187
5	7	0	0	0	0	0	0	0	0	183
6 etc	9	0	0	0	0	1	0	0	0	183

Note that Age in days are not sequential. In this example, no tracked events occurred to any animals of age 4 or 5 days.

Risk is calculated as follows:

For Day 0:

$$Risk_0 = \#\ Births_0 + \#\ AliveIn_0 + \#Arrived_0$$

For subsequent days ($i > 0$):

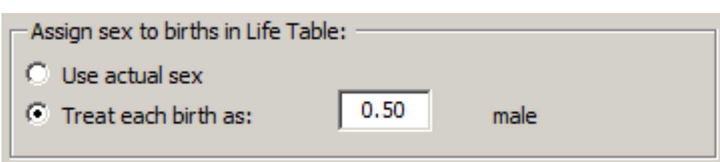
$$R_i = Risk_{i-1} + \#\ Births_i + \#\ + \#AliveIn_i + \#Arrived_i - \#Left_{i-1} - \#AliveOut_{i-1} - \#LTF_{i-1} - \#Died_{i-1}$$

where i = record or row in the frequency data table (not the age in days).

If there are any “out” events for an age (i) where the subsequent age is not i +1 day, then a record is created with the event age of (i+1) so that the “outs” at age i are imposed on the next day rather than the next event which may be several days later than age i.

Reproduction (Calculating M_x)

Reproduction counts are derived from calculating the age of the male and female parent at the time the individual was born and incrementing that age’s “Reproduced” count by .5 in both the male and female frequency table. The incremental value can be changed under the Demographic Settings tab:



When more than one potential male or female parent is listed, the count is incremented depending on the settings for Multiple Parents:

- If set to “Most Likely”: The age of the most likely parent is used or where the most likely parent is the first of equally most likely parents listed
- If set to “Probabilistic parents”: The Reproduced count of the age of each parent listed is incremented by the likelihood that that parent is the parent multiplied by .5 or the user specified incremental value.

If any parent is unknown, the Variable BirthsToUnknownAgedParents is incremented by .5 or the user specified incremental value. If the MultipleParents setting is set to “Probabilistic parents”, then this variable is incremented by the likelihood of the parent multiplied by the incremental value.

The male and female Frequency Tables are printed to the project folder in csv format.

The Raw Life Table

A Raw life table is calculated for males and females from the Frequency Data. This life table provides the life-table rates for day of age during which any event happened. This life-table contains the following columns

Variable	Definition
Age in Days	Age in days
# Deaths	# Deaths of that day age
RiskMortality	From Frequency Table
Q_i	Daily Mortality Rate = # Deaths / # Risk
P_i	Daily Survival Rate = 1 - Q_i
L_i	Survivorship to Day i = l_{i-1} * p_{i-1}
# Births	From Frequency Table
Mi	Fecundity =# Births / # Risk

Example of a Raw Life Table

Age	Deaths	Risk	Qx	Px	Lx	Births	Mx
0	6	192	0.031	0.969	1.000	0	0
1	4	197	0.020	0.980	0.969	0	0
2	5.5	194	0.028	0.972	0.949	0	0
3	1.5	188.5	0.008	0.992	0.922	0	0
4	3	187	0.016	0.984	0.915	0	0
5	1	183	0.006	0.995	0.900	0	0
6	2.5	182	0.014	0.986	0.895	0	0
7	0.5	179.5	0.003	0.997	0.883	0	0
9	2.5	179	0.014	0.986	0.881	0	0
12	1.5	176.5	0.009	0.992	0.868	0	0
13	0.5	175	0.003	0.997	0.861	2	0.011

Raw life tables for males and females are written to csv files in the Project folder.

Creating the Actual and Model Life Tables

The Actual life-table is then derived from the age-in-days raw life table in age classes of a duration defined by the user – the default being yearly age classes of 365 days each - using the following methods where the prime (‘) indicates variables from the daily-raw life table and those without from the User-define age class (UDAC) life-table:

Lx: Let x = age class in UDAC life table (e.g., 0, 1, 2, etc.);
Let z = age in days for the first day of the UDAC (e.g., 0, 365, 730, etc., for annual age classes);
Let y = age in days in the daily raw life table;

If in the raw life-table there is an age that matches the daily age in the UDAC life-table (i.e., z = y) then:

Lx = L’y.

If not, then the z falls between two values of y (e.g., z = 365, but there may only be y values of 360 and 370). In this case let y1 be the y just less than z and y2 the y just greater than z (y1 = 360, y2 = 370):

Let t = (z – y1)/(y2 – y1)

and Lx = `((1 - (t * (1 – P’y1))) * L’y1).`

`Thus, Lx is a linear interpolation between the L’y1 and L’y2.`

M_x: Using the above notation: $M_x = \sum_{z}^{z+1} \mathrm{M'y}$.

That is, M_x is the sum of all the M'_y for those x <= y < (x + d), where d is the duration in days of the UDAC.

$Risk_x$: Using the above notion: $Risk_x = \sum_{z}^{z+1} Risk' / d$ where d is the duration in days of the UDAC.

Thus, the UDAC Risk is the average Risk over the days where x <= y < (x + d).

P_x: $= L_{(x+1)} / L_x$

Q_x: $= 1 – P_x$

MidLx and MidPx are calculated as defined for the raw life tables but using the Lx and Px from the UDAC life table.

Additional life table parameters. The Lx used is the normal Lx for a birth-pulse population and the MidLx for a continuous birth population:

E_x: $= \left(1/L_x \right) \sum_{x}^{\infty} L_t$

C_x: $= {L_x e^{-rx}} / {\sum_0^{\infty} L_x e^{-rx}}$ where r is defined below. Ebert (1999) Equation 2.36

V_x: $= \left({e^{rt}} / {L_x} \right) \sum_{t=x}^{\infty} e^{-rt} L_t M_t$ Ebert (1999) Equation 2.38

The Model life table is a direct copy of the Actual Life table.

Summary Life-Table Statistics:

T (Generation Length) = $\frac{\sum x L_x M_x e^{-rx}}{\sum L_x M_x e^{-rx}}$ Caughley (1977) Pg. 124.

r (intrinsic rate of increase) is calculated using the Newton-Raphson Methods as described in Ebert (1999) pages 16-17.

$\lambda = e^r$.

$R_0 = e^{rT}$.

T, r, λ and R_0 are always presented as yearly rates, regardless of the duration of the User Defined Age Class.n

Standard Error and Confidence Intervals for L_x

In May of 2019, calculations for adding standard errors and confidence intervals for L_x rates were added as follows:

$$S.E.(L_x) \approx L_x \sqrt{\sum_{t=1}^{x} \frac{d_{t-1}}{n_{t-1}(n_{t-1} - d_{t-1})}}$$

Where d_j are the number of deaths in age class j and n_j are the number of individuals at risk for age j.

The approximate 1 – α confidence interval for L_x is:

$$L_x^{\,exp\left[\pm \frac{z_{\alpha/2}}{\ln(L_x)} \sqrt{\sum_{t=1}^{x} \frac{d_{t-1}}{n_{t-1}(n_{t-1} - d_{t-1})}}\right]}$$

Where $z_{\alpha/2}$ is the Standard Normal value for α/2.

Deterministic Projections

The deterministic projections process consists of defining the initial age distribution and applying to it the Mx and Qx rates from the life table for each sex separately to create the age distribution for year 1 and repeating this for subsequent years up to the projection year specified by the user (default set at 20 years).

Defining the Initial Age Structure

The number of individuals in each age class *i* of the initial age distribution (A_0) is calculated with the following steps:

1. Individuals of known sex and age are assigned to the initial age classes:

   $N'_{sx} = N_{sx}$

   Where:
   N' is the number of individuals of sex s and age x in the initial age distribution;
   N is the number of living individuals of known sex *s* and known age class x.

2. Individuals of unknown age but known sex are then added in proportion to the distribution of known age/sex individuals:

   N'_{sx} += $N_{s?} * \left(N_{sx} / \sum_x N_{sx} \right)$

   Where $N_{s?}$ is the number of unknown age but known sex individuals.

3. Individual of known age but unknown sex are then added in proportion to the sex ratio of known sex individuals:

   N'_{sx} += $R * N_{?x}$

   Where $N_{?x}$ is the number of unknown sex individuals of age x; and

   $R = N_{sT} / N_{mT} + N_{fT}$

   Where N_{sT} are the total number of living individual of sex s and N_{mT} and N_{fT} are the total number of males and females.

4. Individuals of unknown age and unknown sex are then added both in proportion to the sex ratio and proportion of living individuals already assigned to age classes:

   N'_{sx} += $(R * N_{??}) * \left(\frac{N'_{sx}}{\sum_i N'_{sx} + N_{s?} + R\,(\sum_i N_{?x})}\right)$

   Where $N_{??}$ is the number of unknown sex and age individuals.

For the initial age structure for the projections of the Stable Age Distribution:

$N'_{sx} = C_{sx} (N_{st} + R\,(N_{?T}))$.

Where N_{sx} is the proportion of a stable population of sex s for age class x and $N_{?T}$ is the total number of unknown sexed individuals.

Projecting an age distribution

Creating the next year's age structure is calculated as follows:

1. Female projections are calculated first. If the life-table is Continuous, then the P_x values used are the MidP_x values. The number in age class x is the N_x from the previous year multiplied by the survival of the previous age class, skipping age class 0:

   $N_{tfx} = N_{(t-1)f(x-1)} * P_{f(x-1)}$ for $x = 1$ to ∞

   Where t is projection year

2. The number of births produced for year t is based on the female M_x values and the number of females at time t:

   $$B = \sum_{x=0}^{\infty} 2 * M_{fx} * N_{tfx}$$

   B is set to 0 if the "Project with no births" is checked.

   If a Reproductive Plan is being applied to births, *B* is defined by the setting on the Reproductive Plan Tab.

3. The number of females in Age class 0 is then calculated as:

   $$N_{tf0} = B * (1.0 - ProportionBirthsMale)$$

   Where *ProportionBirthsMales* is specified in Settings; Default = 0.5

   N_{tf0} is then multiplied by the MidL_0 if the life-table is continuous.

4. The N_{tmx} are calculated in the same was a females except the number of males assigned to the 0 age class is:

   $$N\_tm0 = B * ((ProportionBirthsMale))/ (1.0 - ProportionBirthsMale)$$

   Thus, the male births are a function of the female M_x rates.

5. And Additions or Removals to the population as specified by the settings in the Availability Tab are then applied to the appropriate age classes.

Stochastic Projections

The following steps are taken in this sequence for each simulation:

Defining the Initial Age Structure

Define:
$\alpha(r)$ = 1 if a random uniform number z between 0 and 1 <= r;
= 0 if z > r.

The initial age structures of the males and females are created using the following methods and in this sequence:

1. Individuals of known age and sex are assigned to the appropriate ages in a male or female age structure;

2. Individuals of unknown sex with known age are first assigned a sex randomly in proportion to the sex ratio of the known sexed individuals:

3. Individuals of known sex but unknown age are each randomly assigned an age in proportion to the known age distribution of that sex;

4. Finally, individuals of unknown sex and unknown age are first each randomly assigned a sex in proportion to the known sex ratio, then assigned a age in proportion to the distribution of animals across ages of that sex after step 3 above.

Initial Age Structure for Stable Age Projections

Is this needed?

Projecting the Age Structures

Each year (or UDAC time-step), the following sequence of events takes place:

1. The probability of survival from age x – 1 at time-step *t* - 1 to age x at time step t for each male and female in each age class = $\alpha(P_{s(x-1)})$, where P is the $MidP_x$ if the life-table is continuous.

2. Determine if there are enough breeding-age males to mate with all the females according to the Setting "Maximum number of females bred per male". The number of breeding age females and males is calculated from the living age structure at time-step *t*.

   If the number of breeding males * "Maximum # F/M" < number breeding females, then not all females can be bred. If this is the case, then the specific females that are to be bred (# breeding males * Max F/M) are selected randomly from all breeding-age females, without regards to age. If all females can be bred, all than all breeding-age females are identified as breeders.

3. The total number and sex of births produced during time-step *t*, is the sum of the number of offspring produced by each identified female breeder with the number of offspring being determined by randomly selecting an integer number of offspring from a Poisson distribution with a mean of 2 * M_{fx} where x is the and the sex being assigned randomly in proportion to the sex ration of know sex individuals.

4. For Birth Pulse populations, the number of males and females assigned to age class 0 of time-step t is simply the number of births of each sex. For Continuous life-tables, numbers assigned to age class 0 for each sex $= \sum_1^B \alpha(MidL_{s0})$ where *B* is the number of births of sex *s*.

5. Individuals then are added to or removed from age classes as specified in the settings on the Add/Remove and Availability Tabs in that order.

6. A Census is taken

7. Extinction (only one sex remaining) is determined

8. *r* for males, females and total for time-step t is calculated as $ln(N_t/N_{t-1})$ where the N is the total number of males, females or total.

9. Repeat from Step 1 for the next time-step.